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Observations on the hyperbola $y^2=90x^2+1$

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Abstract
The binary quadratic equation $y^2=90x^2+1$ is considered and a few interesting properties among the solutions are presented. Employing the integral solutions of the equation under consideration, a special Pythagorean triangle is obtained.

Keywords: Binary quadratic, Hyperbola, Integral solutions, Pell equation. 2010 Mathematics subject classification: 11D09

1. Introduction
The binary quadratic equation of the form $y^2 = Dx^2 + 1$, where D is a non-square positive integer, has been studied by various mathematicians for its non-trivial integral solutions when D takes different integral values [1-5]. In [6] infinitely many pythagorean triangle in each of which hypotenuse is four times the product of the generators added with unity are obtained by employing the non-integral solutions of binary quadratic equation $y^2= 3x^2 + 1$. In [7], a special Pythagorean triangle is obtained by employing the integral solutions of $y^2=10x^2+1$. In [8], different patterns of infinitely many pythagorean triangle are obtained by employing the non-trivial solutions of $y^2 = 5x^2 + 1$. In this context one may also refer [9-16]. These results have motivated us to search for the integral solutions of yet another binary quadratic equation $y^2=90x^2+1$ representing a hyperbola. A few interesting properties among the solutions are presented

Notations:

$t_{m,n}$ - Polygonal number of rank n with size m.

p_n^m - Pyramidal number of rank n with size m.

CP_n^m - Centered pyramidal number of rank n with size m.

$CP_{m,n}$ - Centered polygonal number of rank n with size m.

GNO_n - Gnomonic number of rank n.

S_n - Star number of rank n.

2. Method of Analysis:
The binary quadratic equation representing hyperbola under consideration is

$$y^2 = 90x^2 + 1 \tag{1}$$

whose general solution (x_n, y_n) is given by

$$x_n = \frac{g}{2\sqrt{90}}, \quad y_n = \frac{f}{2}$$

where
$$f = (19 + 2\sqrt{90})^{n+1} + (19 - 2\sqrt{90})^{n+1}$$
$$g = (19 + 2\sqrt{90})^{n+1} - (19 - 2\sqrt{90})^{n+1}$$
, n=0,1,2,3,……

The recurrence relations satisfied by x and y are given by

$$y_{n+2} - 38y_{n+1} + y_n = 0, y_0 = 19, y_1 = 721$$

$$x_{n+2} - 38x_{n+1} + x_n = 0, x_0 = 2, x_1 = 76$$

Some numerical examples of x and y satisfying (1) are given in the following table:

n	x_n	y_n
0	2	19
1	76	721
2	2886	27379
3	109592	1039681
4	4161610	39480499
5	1580315880	1499219281

From the above table, we observe some interesting properties:

1. x_n is always even
2. y_n is always add
3. $y_{2n} \equiv 0$ (mod 19)
4. $x_{2n+1} \equiv 0$ (mode 76)
5. $2y_{2n+1} + 2$ is a perfect square
6. $6(2y_{2n+1} + 2)$ is a nasty number
7. $y_{3n+2} + 3fn$ is a cubic integer
8. $y_{n+1} = 19y_n + 180x_n$
9. $y_{n+2} = 721y_n + 6840x_n$
10. $x_{n+1} = 19x_n + 2y_n$
11. $x_{n+2} = 721x_n + 76y_n$
12. $y_{3n+2} + 3y_n = 2y_n(y_{2n+1} + 1)$
13. $(y_{3n+2} + 3y_n)^2 = 16y_n^6$
14. $19y_{n+2} - 721y_{n+1} = 180x_n$
15. $6840y_{n+1} - 180y_{n+2} = 180y_n$
16. $19x_{n+1} - 2y_{n+1} = x_n$
17. $19y_{n+1} - 180x_{n+1} = y_n$
18. $19x_{n+2} - 76y_{n+1} = 19x_n$
19. $721y_{n+1} - 180x_{n+2} = 19y_n$
20. $721x_{n+1} - 2y_{n+2} = 19x_n$
21. $19y_{n+2} - 6840x_{n+1} = 19y_n$
22. $721x_{n+2} - 76y_{n+2} = x_n$
23. $721y_{n+2} - 6840x_{n+2} = y_n$
24. $19x_{n+2} - 721x_{n+1} = 2y_n$
25. $76x_{n+1} - 2x_{n+2} = 2x_n$
26. $19y_n y_{n+2} - 721y_n y_{n+1} = 180x_n y_n$
27. $6840x_n y_{n+1} - 180x_n y_{n+2} = 180x_n y_n$
28. $x_n y_{n+1} - y_n x_{n+1} = 180x_n^2 - 2y_n^2$
29. $y_n y_{n+1} - 90x_n x_{n+1} = 19y_n^2 - 1710x_n^2$
30. $y_n x_{n+2} - 4y_n y_{n+1} = x_n y_n$
31. $721x_n y_{n+1} - 180x_n x_{n+2} = 19x_n y_n$
32. $x_n y_{n+2} - y_n y_{n+2} = 6840x_n^2 - 76y_n^2$
33. $y_n y_{n+2} - 90x_n x_{n+2} = 721y_n^2 - 64890x_n^2$
34. $4y_n x_{n+1} - x_n x_{n+2} = 8y_n^2 - 721x_n^2$
35. $72x_n x_{n+1} - 2y_n x_{n+2} = 13699x_n^2 - 152y_n^2$
36. $S_f = 6[38(y_{2n+2} - y_{n+1}) - 360(x_{2n+2} - x_{n+1})] + 13$
37. $GNO_f = 76y_{n+1} - 720x_{n+1} - 1$
38. $6P_f^m = 38[(m-2)y_{3n+3} + 3y_{2n+2} + (2m-1)y_{n+1}]$ $-360[(m-2)x_{3n+3} + 3x_{2n+2} + (2m-1)x_{n+1}] + 6$
39. $6CP_f^m = 38[m(y_{3n+3} + 2y_{n+1} + y_{n+1})] - 360[m(x_{3n+3} + 2x_{n+}$
40. $2t_{m,f} = 38[(m-2)y_{2n+2} - y_{n+1}(m-4)] - 360[(m-2)x_{2n+2}$
41. $2CP_{m,f} = m[(m-2)(38y_{2n+2} - 360x_{2n+2}) - (m-4)(38y_{n+1}$

Remarkable Observations:

1. Let, $y = 38y_{n+1} - 360x_{n+1} and x = 19x_{n+1} - 2y_{n+1}$ and. Then the pair (x, y) satisfies the hyperbola $y^2 = 360x^2 + 4$.
2. Let m, n be two non-zero distinct positive integers such that

$$m = x_n + 2y_n, n = x_n ,$$

Note that m $m > n > 0$. Treat m, n as the generators of the Pythagorean triangle $T(\alpha, \beta, \gamma)$.

where $\alpha = 2mn$, $\beta = m^2 - n^2$, $\gamma = m^2 + n^2$

Let A, P represent the area and perimeter of $T(\alpha, \beta, \gamma)$.

Then the following interesting relations are observed:

(1) $\alpha + 44\lambda - 45\beta + 1 = 0$

(2) $46\beta - 45\gamma - 4\frac{A}{P} = 1$

(3) $\gamma - 46\alpha - 180\frac{A}{P} = 1 Ro$

3. Conclusion:

To conclude, one may search for other choices of hyperbola for patterns of solutions and their corresponding properties.

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