International Journal of Multidisciplinary Research and Development

Volume: 2, Issue: 9, 304-305
Sep 2015
www.allsubjectjournal.com
e-ISSN: 2349-4182
p-ISSN: 2349-5979
Impact Factor: 4.342

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Observations on the Cone $z^2 = ax^2 + a(a-1)y^2$

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Abstract
An attempt has been made to obtain all possible integer solutions satisfying the cone given by $z^2 = ax^2 + a(a-1)y^2, a > 1$

Keywords: Quadratic, Rational solutions, Integer solutions
2010 MSC Classification: 11D09

Introduction

Ternary quadratic Diophantine equations representing cone are rich in variety and one may refer [1-5] for the non-zero distinct integer solutions satisfying the respective equation. In the above references, the integer solutions are obtained by applying the factorization method and employing the method of cross multiplication.
In some cases, the linear transformations have been introduced to obtain the integral solutions for the respective equation. It is observed that one may obtain all possible integer solutions through a single process which is the main thrust of this process.

2. Method of Analysis

Ternary quadratic Diophantine equation to be solved is

$$z^2 = ax^2 + a(a-1)y^2, a > 1 \qquad (1)$$

Assume $z = aw$ (2)

Substituting (2) in (1), we get

$$aw^2 = x^2 + (a-1)y^2 \qquad (3)$$

Consider $\frac{x}{w} = X, \frac{y}{w} = Y$ in (3)

Then, $(3) \Rightarrow X^2 + (a-1)Y^2 = a$ (4)

Where X and Y are rational
Considering the linear transformation

$$Y = t(X-1) + 1 \qquad (5)$$

In (4), it leads to

$$[1 + (a-1)t^2]X^2 + [(2t - 2t^2)(a-1)]X + (a-1)(t-1)^2 - a = 0$$

Where t is rational
Treating (6) as quadratic in X and solving for X, we have

$$X=\frac{1}{2[1+(a-1)t^2]}[(2t^2-2t)(a-1)\pm$$
$$\sqrt{(a-1)^2 4t^2(t-1)^2-4[1+(a-1)t^2][(a-1)(t-1)^2-a]}$$
$$=\frac{1}{1+(a-1)t^2}[t(t-1)(a-1)\pm((a-1)t+1)]$$
$$X=1,\frac{(a-1)t^2-2(a-1)t-1}{(a-1)t^2+1}$$

Substituting the X values in (5), we get the corresponding Y values to be

$$Y=1,\frac{1-2t-(a-1)t^2}{(a-1)t^2+1}$$

The values X=Y=1 lead to the integer solutions of (1) to be (w,w,aw)

Considering the rational values of X, Y, note that for any given ‘a’, it is possible to choose rational‘t’ and integer ‘w’ so that x,y,z are integers.

A few numerical examples are presented below

a	t	w	x	y	z
5	$\frac{1}{2}$	4	-8	-2	20
4	$\frac{1}{3}$	5	-10	0	20
2	$\frac{1}{2}$	10	-14	-2	20
3	$\frac{1}{4}$	9	-15	3	27
6	$\frac{1}{5}$	3	-7	1	18

For simplicity, choosing’ w’ as $w=\alpha[1+(a-1)t^2],\alpha\in z-\{0\}$, the integer solutions of (1) are given by

$$x=\alpha[(a-1)t^2-2(a-1)t-1]$$
$$y=\alpha[1-2t-(a-1)t^2]$$
$$z=a\alpha[1+(a-1)t^2]$$

3. Conclusion

In this paper, we have made an attempt to obtain all integer solutions through a single process. To conclude, one may search for the equations for which the above method is applicable.

4. Acknowledgement

*The finicial support from the UCG, New Delhi (F-MRP-5122/14(SERO/UCG) dated March 2014) for a part of this work is gratefully acknowledged.

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