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Content Knowledge and Pedagogical Content Knowledge in the seventh grade mathematics textbook of West Bengal Board of Secondary Education

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Abstract
Present work deals with an analysis of content knowledge and pedagogical content knowledge in seventh grade mathematics textbook of West Bengal Board of Secondary Education, West Bengal, India. Concepts which are explored in the algebra together with necessary concepts which should be included are also discussed. An analysis on textual presentation and exercises supplied with necessary recommendations are argued.

Keywords: Mathematics textbook, Algebra, Content knowledge, Pedagogical Content Knowledge.

Introduction

Shulman (1987) defines seven categories to provide a framework for teacher knowledge which are:

1. Content knowledge
2. General pedagogical knowledge eg classroom control, using group work
3. Pedagogical content knowledge
4. Curriculum knowledge
5. Knowledge of learners and their characteristics
6. Knowledge of educational contexts eg schools and the wider community
7. Knowledge of educational ends purposes and values

Shulman (1987) identified seven domains of teacher knowledge, one of which is pedagogical content knowledge. He explained why he identified pedagogical content knowledge as a knowledge domain for teachers as follows:

Pedagogical content knowledge is of special interest because it identifies the distinctive bodies of knowledge for teaching. It represents the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented and adapted to the diverse interests and abilities of learners, and presented for instruction.

Pedagogical content knowledge is the category most likely to distinguish the understanding of the content specialist from that of the pedagogue. (p. 8) Shulman claimed that pedagogical content knowledge is a distinct body of knowledge even though knowledge of content and knowledge of pedagogy contribute to it. He also noted that pedagogical content knowledge includes knowledge of learners, knowledge of educational context, and knowledge of instructional materials.

Tamir (1988) [13] made a distinction between general pedagogical knowledge and subject-matter-specific pedagogical knowledge. He claimed that each type of knowledge is composed of four categories-namely, student, curriculum, instruction, and evaluation- but they have different meanings in each domain. He also identified teachers' skills in diagnosing students' conceptual difficulties in a given topic and their knowledge about effective use of instructional tools as subject-matter-specific pedagogical knowledge.

Ball and Bass (2000) [1] identified teachers' knowledge of students' difficulties and appropriate teaching strategies to eliminate those difficulties as part of teachers' pedagogical content knowledge. They defined pedagogical content knowledge as follows:

'Pedagogical content knowledge is a special form of knowledge that bundles mathematical knowledge with knowledge of learners, learning, and pedagogy. These bundles offer a crucial resource for teaching mathematics, for they can help the teacher anticipate what students might have trouble learning, and have ready alternative models or explanations to mediate those difficulties (p. 88).'

Wang Wei Sönnerhed (2011) [15] studied algebra textbook for CK and PCK and wrote 'The primary aim of the study is to explore what pedagogical content knowledge regarding solving quadratic equations that is embedded in mathematics textbooks. The secondary aim is to analyze the algebra content related to solving quadratic equations from the perspective of mathematics as a discipline in relation to algebra history. It is about what one can find in the textbook rather than how the textbook is used in the classroom (p-5).'

Sen and Samanta (2015) [11] analyzed content knowledge and pedagogical content knowledge in sixth grade mathematics textbook of West Bengal Board of Secondary Education, West Bengal, India. According to them 'It seems that the algebra content of the text book demands slide modifications. Lastly it is suggested that there should be a concept summary listed at the end of the text and an exercise containing problems on every concept at the extreme end of the units discussed before for correlation and evaluation of concepts.'

Methodology

To explore the nature of Content Knowledge (CK) represented in the textbook and expected Pedagogical Content Knowledge (PCK) in the framework, it is necessary to discuss Van Dormolin's (1986) classification of teaching perspectives and learning perspectives of Schmidt *et al* (1997) [10].

Based on their classification and the CK-PCK overall framework one may consider the following criteria for analyzing algebra content textual presentation as follows:

1. Consistency and clearness of Mathematical content: A mathematical text should be consistent and clear to the reader. "There must be no errors, either of computation or of logic. Proofs might be incomplete, but not false. Conventions must be used consistently. […] the content must be clear to the intended reader." (Van Dormolen, 1986, p. 151).
2. Mathematical theoretical aspects: This criterion concerns knowledge elements such as mathematical theorems, rules, definitions, methods and conventions. Such mathematical knowledge is called "kernels" (Van Dormolen, 1986, p. 146)
3. Mathematical content development and connections: This criterion is based on the classification of Schmidt *et al*. (1997) [10]. By means of this criterion, one may investigate how mathematical content topics relate to each other in the chapter of algebra. The aim is to explore the embedded teaching trajectory related to text.
4. Mathematical representations and applications: This category often reflects different views. A formalistic view regards mathematics as a set of concepts, rules, theorems and structures. Mathematics applications are often regarded as informal view. In an informal view students are encouraged to engage in activities like generalizing, classifying, formalizing, ordering, abstracting, exploring patterns and so on, and new ideas are encouraged (De Lange, 1996; Freudenthal, 1991; Goldin, 2008; Pepin *et al*., 2001; Van Dormolen, 1986; Vergnaud, 1987) [4, 5, 6, 9, 14].
5. Language use: In which way are mathematical theorems, definitions, and rules explained and illustrated: formally in a mathematical language or pedagogically in combination with everyday language, in order to make sense for a student reader.
6. To analyze different kinds of mathematics exercises, activities and problems as well as tests in the textbook, it is important to analyzing mathematics tasks in the textbooks (Brändström, 2005) [2]. One may consider the following points:

A. Routine exercises refer to the kind of exercises that require students to use newly presented mathematical concepts, rules or algorithmic procedures illustrated in examples, in order to get familiar with the content. This kind of exercises is often at a basic level and requires simple and similar operations or reasoning to those just presented.

B. Exercises that require students to evaluate, analyze and reason mathematically instead of merely computing mechanically (Brändström, 2005) [2]. Such exercises intend to encourage students to understand the integration of mathematics concepts and procedures (Hiebert & Carpenter, 2007; Hiebert & Lefevre, 1986) [7, 8].

C. Exercises that are related to real world contexts. Such exercises are often word problems (or called real world problems) and the pedagogical reason of using them is to bring reality into the mathematics classroom, to create occasions for learning and practicing the different aspects of applied problem solving without the practical contact with the real world situation (Chapman, 2006) [3]. They reflect the view of mathematics applications in real-life situations (De Lange, 1996; Freudenthal, 1991; Goldin, 2008; Pepin *et al*., 2001; Van Dormolen, 1986; Vergnaud, 1987) [4, 5, 6, 9, 14].

Name of the seventh grade mathematics text book of West Bengal Board of Secondary Education is 'Ganitprava (class VII)'. In this book, content of unit 6 (pages 77-109) is Algebraic Processes.

The concepts which are explored in this chapter are

- ✓ Factors of monomials and term wise factors of binomial expressions are discussed. For example 3x+1, 6p-1 and $5x^2$ +y are considered and term wise factors are illustrated.
- ✓ Concept of terms of an algebraic expression are introduced.
- ✓ Concept of factors of a monomial is expressed as sides of rectangle or square. For example 2xy = 2x.y can be represented by a rectangle whose adjacent sides are 2x and y.
- ✓ Concept of polynomial is introduced by increasing terms as monomial, binomial, trinomial etc.
- ✓ After that, concepts of similar and dissimilar terms are introduced.
- ✓ Concept of expressing polynomial by colour cards is introduced. For example expressing $3x^2$ +4x+6 a specific length is used for variable x and another length for 1. Hence 'x^2' is represented by a square, 'x=x.1' by a rectangle and '1=1.1' by a square. Finally, '$3x^2$'represented by three square with side 'x', '4x' represented by four rectangle with adjacent sides x and 1, 6 represented by six square with side '1'. Here three colours are used for representing x^2, x and 1.
- ✓ Term wise addition is illustrated by activity. For example it is shown that $(2x^2+3x+5) + (3x^2+4x+6) \equiv (2+3)$ square with x + (3+4) rectangle with adjacent sides x and 1+ (5+6) square with side 1.
- ✓ For subtraction, each card is coloured by blue in one face to represent positive sign and red on opposite face to represent negative sign. Initially cards are arranged according to their sign and in the time of subtraction, faces of the cards of the polynomial which is subtracted is reversed.

- ✓ Next, concept of formation of polynomial with the help of everyday problems is explored. Here distributive law of multiplication over addition is used to add similar terms of a polynomial. For example x+x=1.x+1.x=(1+1)x=2x.
- ✓ Concept of addition of similar terms is defined as addition of coefficients of variable quantity and dissimilar quantities are listed with '+' or '-' sign respectively.
- ✓ Concept of subtraction is represented with the help of addition of negative of the polynomial which is to be subtracted with the help of distributive law.
- ✓ Concept of addition and subtraction by traditional method is presented (p-93).
- ✓ Next concept is determination of value of polynomials for given values of indefinite number. To explore those concept activities of different pattern formation by match sticks is considered.
- ✓ To explore multiplication activity of arrangement of buttons is considered. Here array type arrangement of rows and columns are explored. Starting with concrete numbers, abstract numbers are introduced to count total number of elements present in the array. Also in an example aria is represented by multiplication of length and breadth.
- ✓ Next concept 'multiplication is distributive over addition' is presented by practical verification with colour pitch board.
- ✓ Use of 'distributive law of multiplication over addition' is explored by realistic examples. Also an attempt is made to simplify algebraic expressions.
- ✓ Concept of multiplication of binomial by binomial, polynomial by binomial etc. are explored.
- ✓ Division is initially expressed as-- if a number is expressed as product of two numbers then $A = B\times C \Rightarrow A\div B = C \; and \; A\div C = B$.
- ✓ Concept of 'any number ÷ 0 is undefined' is discussed with the help of 'repeated subtraction ≡division'.
- ✓ Some routine examples are also incorporated on division with the help of index rule $\frac{a^m}{a^n} = a^{m-n} \; (a \neq 0)$, an alternative process is also discussed by cancelling some terms from numerator and denominator of a fraction.
- ✓ To extend the concept of division $(A+B)\div C = \frac{A+B}{C} = \frac{A}{C} + \frac{B}{C} \; (C \neq 0)$ is illustrated with examples. It is also extended to $(A+B+C)\div X = \frac{A+B+C}{X} = \frac{A}{X} + \frac{B}{X} + \frac{C}{X} \; (X \neq 0)$

There are some suggestions for improvement of the text as follows

- ➢ It should be clear that term wise factors are not necessarily the factors of the binomials. For example, factors of 3x, 6p, $5x^2$ and y respectively are not the factors of 3x+1, 6p-1 and $5x^2$ +y.
- ➢ There is no discussion in the text that 'coefficient' and 'numerical coefficient'. In the text only the words 'constant' and 'coefficient' are used but in the exercise 4 (page-85) the term 'numerical coefficient' is used. There is a chance of confusion between 'coefficient' and 'numerical coefficient'.
- ➢ Activity for addition is explored by different size and colour of card board representing x^2, x and 1. It is my suggestion that use different size for representing x^2, x and 1 but use one specific colour for positive term and another specific colour for negative terms so that it will be compatible for subtraction also. For example use either green or blue to represent positive quantities and red for negative quantities. It will be systematic for realization of operation. Otherwise if one wants to add $3x^2$ +3x-5 and $2x^2$ -4x+6 it may rise a problem.
- ➢ For the concept 'factors of monomial can be expressed as sides of a rectangle' one can rewrite 2xy = x.2y in that case rectangle can be constructed whose adjacent sides are x and 2y and so on should be discussed.
- ➢ Generalizations like

$$x\times(a+b+c+..........to\, n\, terms) =$$
$$x\times a + x\times b + x\times c + ...n\, terms$$

$$(a+b+c+..........to\, n\, terms)\div x =$$
$$\frac{a}{x} + \frac{b}{x} + \frac{c}{x} + ...to\, n\, terms$$

Should be explored.

To analyze the mathematical exercise represented in this unit it is observed that

1. Routine exercises are included.
2. Exercises are arranged in such a way that can evaluate, analyze the concepts of the students. Also these exercises help to increase the power of mathematical reasoning of the learner.
3. Questions are represented according to the structure of the text and these are sequentially arranged.
4. Most of the problems are chosen from everyday life of the learner and common people.

Let us consider the unit 12 Algebraic Theorems (Pages 146-167). The concepts which are explored in this chapter are:

- ✓ Concept of verification of $(a+b)^2$ numerically and geometrically is described.
- ✓ Proof of $(a+b)^2 = a^2 + 2ab + b^2$ by multiplication is described.
- ✓ Proof of $(a-b)^2 = a^2 - 2ab + b^2$ by multiplication is established.
- ✓ Geometric and numerical verification of $(a-b)^2$ is discussed.
- ✓ Concept of identity is explored and it is also mentioned that above mentioned theorems are identities.
- ✓ Three results $(a+b)^2 + (a-b)^2 = 2(a^2+b^2)$, $(a+b)^2 - (a-b)^2 = 4ab$ and $ab = \left(\frac{a+b}{2}\right) - \left(\frac{a-b}{2}\right)$ are deduced.
- ✓ An extension of the theorem $(a+b)^2$ to $(a+b+c)^2$ is shown.
- ✓ Different types of examples are also discussed by putting different values of a and b.
- ✓ It is discussed that $(a+b)^2 = a^2 + 2ab + b^2$, $(a-b)^2 = a^2 - 2ab + b^2$, $(a+b)^2 + (a-b)^2 = 2(a^2+b^2)$,

$(a+b)^2-(a-b)^2=4ab$, $ab=\left(\frac{a+b}{2}\right)-\left(\frac{a-b}{2}\right)$ and $(x+a)(x+b)=x^2+x(a+b)+ab$ are all identities.

- ✓ Rearrangement of $(a+b)^2=a^2+2ab+b^2$ and $(a-b)^2=a^2-2ab+b^2$ gives respectively $a^2+b^2=(a+b)^2-2ab$ and $a^2+b^2=(a-b)^2+2ab$ is discussed.
- ✓ Examples related to those theorems are also illustrated.
- ✓ Geometric verification, algebraic proof and numerical verification of the theorem $a^2-b^2=(a+b)(a-b)$ is shown.
- ✓ Recapitulation of the theorems $(a+b)^2$, $(a-b)^2$, a^2-b^2 and important results is done and suitable examples are also illustrated.

It is suggested that at least three sets of numbers for a and b satisfies the theorems to discuss the understand identity. In the text it is only mentioned that they are identities but for clear understanding there should be verification and exercise.
There should be a discussion about the cyclic order for remembering the result of $(a+b+c)^2$.
Another extension of $(a+b)^2$ i.e. $(a+b+c+d)^2$ may be incorporated by writing $(x+y)^2=x^2+2xy+y^2$ and putting $x=a+b$ and $y=c+d$.

One may analyze the mathematical exercise represented in this unit as

1. Routine exercises are included but exercise about the verification of identities should be incorporated. There is no exercise about the verification of identity.
2. Exercises are arranged in such a way that can evaluate, analyze the concepts of the students. Also these exercises help to increase the power of mathematical reasoning of the learner.
3. Questions are represented according to the structure of the text and these are sequentially arranged.
4. Most of the problems are chosen from everyday life of the learner and common people.

Another unit (19) represents Factorization (pages 219-228). The concepts which are explored in this chapter are

- ✓ Concept of factors and prime factors of a number (18, 20) explored with the help of trees.
- ✓ Next concept of factorization of algebraic expressions like $2xy, 4xz, 9x^2y, 2x(x+4)$ and $8x^2y^2(x+2y)$ is analyzed.
- ✓ A statement is made that prime factors are irreducible.
- ✓ Factors of $3x^2+6x$ is shown by activity method.
- ✓ Concept of getting the algebraic expressions with the help of factors is illustrated by trees.
- ✓ Some examples and exercises are illustrated and listed respectively for application of the concept of factorization.
- ✓ Application of algebraic identities $a^2+2ab+b^2=(a+b)^2$, $a^2-2ab+b^2=(a-b)^2$, $a^2-b^2=(a+b)(a-b)$, $a^2+b^2=(a+b)^2-2ab$ for factorization is presented.

Examples are well arranged and according to the concepts presented in the text.
To analyze the mathematical exercise represented in this unit it is observed that

1. Routine exercises are included.
2. Exercises are arranged in such a way that can evaluate, analyze the concepts of the students. Also these exercises help to increase the power of mathematical reasoning of the learner.
3. Questions are represented according to the structure of the text and these are sequentially arranged.
4. Most of the problems are chosen from everyday life of the learner and common people.

Lastly, the unit 22 which deals with Construction of equation and solution (pages 250-265) is considered. Concepts which are explored are given below:

- ✓ Concept equation through activity by match sticks is explored.
- ✓ Concept of root of an equation by trial and error method is established.
- ✓ Concept of linear equation of one variable is illustrated.
- ✓ Construction of equation with different conditions is explored.
- ✓ Rule of addition, subtraction, division (by non-zero number) and multiplication of same number with both sides of an equation is explained by realistic example by balance.
- ✓ By using above mentioned rule solution of equation is illustrated.
- ✓ Construction and solution of equations in real life problems is discussed by suitable examples.

Examples from real life are considered to correlate theory and application in real world.
To analyze the mathematical exercise represented in this unit it is observed that

1. Routine exercises are included.
2. Exercises are arranged in such a way that can evaluate, analyze the concepts of the students to increase the power of mathematical reasoning.
3. Questions are represented according to the structure of the text and these are sequentially arranged.
4. Real life problems are incorporated.

Concluding remarks

It is mentioned in page number 287 of the text book that the book is written on the basis of NCF 2005 and learning will be activity based. It is actually psychological to logical approach to relate mathematics to real world. Teachers are encouraged to help in constructing students' knowledge (p-287-288). Examples are realistic and appropriate for class VII. Principles for exploration of subject matter are simple to complex, real to abstract and known to unknown. Activity based teaching strategy is preferred. Analysis and Synthesis method together with Discovery and Problem Solving may be a suitable combination for instruction. It seems that the algebra content of the text book demands slide modifications. Lastly it is suggested that there should be a concept summary listed at the end of the text also an exercise containing problems on every concept at the extreme end of the units for correlation and evaluation of concepts.

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