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On Soft-slightly-GSR-continuous functions

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Abstract
In this paper, we introduce and study the concept of soft slightly-gsr-continuous functions which is weaker than soft-gsr-continuous functions space. The relationship between soft slightly-gsr-continuity and other related functions is also analyzed. Furthur, we discuss soft-gsr-regular space.

Keywords: soft- gsr-closed set, soft slightly-gsr-continuity, soft-gsr-regular space

1. Introduction

Molodtsov [7] initiated the concept of soft set theory as a new mathematical tool and presented the fundamental results of the soft sets. Muhammad Shabir and Munazza Naz [4] introduced soft topological spaces and the notions of soft open sets, soft closed sets, soft closure, soft interior points, soft neighborhood of a point and soft separation aioms. Kharal *et al.* [16] introduced soft function over classes of soft sets. In 2013, Bin Chen [8, 14] introduced and explored the properties of soft semi open sets and soft semi closed sets in soft topological spaces. Soft regular open and soft regular closed sets are introduced by Saziye Yuksel [12]. Cigdem Gunduz Aras *et al.*, [3] in 2013 studied and discussed the properties of Soft continuous mappings which are defined over an initial universe set with a fixed set of parameters. In this paper, we introduce and study the concept of soft slightly- gsr-continuous functions which is weaker than soft -gsr-continuous functions and also we study soft-gsr-regular space and obtain their fundamental properties.

2. Preliminaries

Definition 2.1: [4, 6, 7, 8] Let U be the initial universe and P(U) denote the power set of U. Let E denote the set of all parameters. Let A be a non-empty subset of E. A pair (F, A) is called a soft set over U, where F is a mapping given by F: A→P(U). In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon \in A$, F(ε) may be considered as the set ε- approximate elements of the soft set (F,A).Clearly, a soft set is not a set. Two soft sets (F,A) and (G,B) over a common universe U are said to be soft equal if (F,A) is a soft subset of (G,B) and (G,B) is a soft subset of (F,A).

Definition 2.2: [4, 6, 7, 8] For a soft set (F, A) over the universe U, the relative complement of (F, A) is denoted by $(F,A)^c$ and is defined by $(F,A)^c = (F^c, A)$, where F^c: A → P(U) is a mapping defined by F^c(e) = U - F(e) for all e $\in$ A.

Definition 2.3: [2] A soft set (F,A) over U is said to be Null soft set denoted by ϕ if ∀e ∈ A, F(e) = ϕ.
Absolute soft set denoted by A, if ∀e ∈ A, F(e) = U.

Definition 2.4: [4, 6, 7, 8] The union of two soft sets of (F,A) and (G,B) over the common universe U is soft set (H,C), where C = A $\cup$ B and for all e ε C, H(e) = F(e) if e ε A - B, H(e) = G(e) if e ε B-A and H(e)=F(e) $\cup$ G(e) if e ε A $\cap$ B. We write (F,A) $\cup$ (G,B)= (H,C).
The Intersection (H,C) of two soft sets (F,A) and (G,B) over a common universe U denoted (F,A) $\cap$ (G,B) is defined as C= A $\cap$ B and H(e) = F(e) $\cap$ G(e) for all e ε C.

Definition 2.5: [4] Let τ be the collection of soft sets over X, then τ is called a soft topology on X if τ satisfies the following axioms:
1)ϕ, $\widetilde{X}$ belong to τ.
2) The union of any number of soft sets in τ belongs to τ.
3) The intersection of any two soft sets in τ belongs to τ.
The triplet (X, τ, E) is called a soft topological space over X. For simplicity, we can take the soft topological space (X, τ, E) as X throughout the work.

Definition 2.6: [4] Let (X, τ, E) be soft space over X. A soft set (F, E) over X is said to be soft closed in X, if its relative complement $(F,E)^c$ belongs to τ. The relative complement is a mapping F^c : E→P(X) defined by F^c(e) = X - F(e) for all e $\in$ A.

Definition 2.7: [4] Let X be an initial universe set, E be the set of parameters and τ = {ϕ, $\widetilde{X}$ }. Then τ is called the soft indiscrete topology on X and (X, τ, E) is said to be a soft indiscrete space over X. If τ is the collection of all soft sets which can be defined over X, then τ is called the soft discrete topology on X and (X, τ, E) is said to be a soft discrete space over X.

Definition 2.8: [4] Let (X, τ, E) be a soft topological space over X and the soft interior of (F,E) denoted by Int(F,E) is the union of all soft open subsets of (F,E). Clearly, (F,E) is the largest soft open set over X which is contained in (F,E). The soft closure of (F, E) denoted by Cl(F,E) is the intersection of all closed sets containing (F,E). Clearly, (F, E) is smallest soft closed set containing (F,E).

Int (F, E) = $\cup$\{ (O,E): (O,E) is soft open and (O,E) $\tilde{\subseteq}$ (F,E)\}.

Cl(F,E) = $\cap$\{ (O,E): (O,E) is soft closed and (F,E) $\tilde{\subseteq}$ (O,E)\}.

Definition 2.9: [7] Let (F, E) be a soft set over X. The soft set (F, E) is called soft point, denoted by (x_e, E), if for element e ∈ E, F(e) = \{x\} and F(e') = ∅ for all e'∈ E−\{e\}.

Definition 2.10: [8, 9, 10] A soft subset (A, E) of X is called
1. A soft generalized closed (Soft g-closed) in a soft topological space (X, τ, E) if Cl(A,E) $\tilde{\subseteq}$ (U,E) whenever (A,E) $\tilde{\subseteq}$ (U,E) and (U,E) is soft open in X.
2. A soft semi open if (A,E) $\tilde{\subseteq}$ Int(Cl(A,E))
3. A soft regular open if (A,E)= Int(Cl(A,E)).
4. A soft clopen is (A, E) is both soft open and soft closed.

The complement of the soft semi open, soft regular open, soft α-open, soft b-open, soft pre-open sets are their respective soft semi closed, soft regular closed, soft α-closed, soft b-closed and soft pre -closed sets.

The finite union of soft regular open sets is called soft π -open set and its complement is soft π-closed set. The soft regular open set of X is denoted by SRO(X) or SRO(X, τ, E).

Definition 2.11: [8, 14] The soft semi closure of (A, E) is the intersection of all soft semi closed sets containing (A, E). (i.e) The smallest soft semi closed set containing (A, E) and is denoted by sscl (A, E). The soft semi interior of (A, E) is the union of all soft semi open set contained in (A, E) and is denoted by ssint (A, E).

Similarly, we define soft regular-closure, soft α-closure, soft pre-closure, soft semi closure and soft b-closure of the soft set (A,E) of a topological space X and are denoted by srcl(A, E), sαcl(A,E), spcl (A,E), sscl (A, E) and sbcl (A, E) respectively.

Definition 2.12: [11, 13] Let (X, τ,E) and (Y, τ ′,E) be two soft topological spaces. A function f : (X, τ, E) → (Y, τ′,E) is said to be

(i) Soft semi-continuous if f^{-1} (G,E) is soft semi-open in (X, τ, E), for every soft open set (G,E) of (Y, τ ′,E).

(ii) Soft regular-continuous if f^{-1}(G,E) is soft regular-open in (X, τ,E), for every soft open set (G,E) of (Y, τ ′,E).

Definition 2.13: [6] Let (X, τ, A) and (Y, τ*,B) be soft topological spaces and f_{pu}: SS(X) $_A$→ SS(Y)$_B$ be a function. Then the function f_{pu} is called soft open mapping if f_{pu} (G, A) ∈ τ* for all (G, A)∈ τ. Similarly, a function f_{pu}: SS(X)$_A$→SS(Y)$_B$ is called a soft closed map if for a closed set (F,A) in τ, the image f_{pu} (G,B) is soft closed in τ*.

Definition 2.14: [5] A soft topological space X is called soft -gsr- $T_{1/2}$ space if every soft- gsr- closed set is soft semi-closed

Definition 2.15: [4] A soft topological space (X, τ, E) is a soft -T_0 space, if for each pair of distinct soft points x and y in X, there exist soft open sets (F,E) and (G,E) such that x∈ (F,E) and y∉ (F, E) or y∈ (G,E) and x∉ (G, E).

Definition 2.16: [5] A soft subset (A, E) of a soft topological space X is called soft-gsr-closed set in X if sscl (A, E) $\tilde{\subseteq}$ (U, E) whenever (A, E) $\tilde{\subseteq}$ (U, E) and (U, E) is soft regular open in X.

Definition 2.17: [5] The Soft -gsr-Closure of a soft set (G, E) is defined to be the intersection of all soft -gsr-closed sets containing the soft set (G, E) and is denoted by sgsr-cl(G, E).
The Soft- gsr-Interior of a soft set (G, E) is defined to be the union of all soft -gsr-open sets contained the soft set (G, E) and is denoted by sgsr-int (G, E).

Definition 2.18: [15] A space (X, τ, E) is said to be soft locally indiscrete, if every soft open set of X is soft closed in X.

Definition 2.19: [15] A function f: (X, τ, E) → (Y, τ′, E) is called soft slightly continuous, if f^{-1} (G, E) is soft open in X for each soft clopen subset (G, E) of Y.

Definition 2.20: [4] Let (X,$\tilde{\tau}$) be a soft topological space over X, (G,E) be a soft closed set in X and x ∈ X such that x ∉ (G, E). If there exist soft open sets (F_1, E) and (F_2,E) such that x ∈ (F_1, E),(G, E) $\subseteq$ (F_2, E) and (F_1, E) $\cap$ (F_2, E) = ϕ, then (X,$\tilde{\tau}$) is called a soft- regular space

3. Slightly- gsr - continuous functions

Definition 3.1: A function f : (X,τ,E) → (Y,τ′,E) is said to be soft slightly- gsr -continuous, if f^{-1} (G, E) is - gsr-open in X for each soft clopen subset (G, E) of Y.

Theorem 3.2: The following statements are equivalent for a function f: (X,τ,E) → (Y,τ′,E)
1. F is soft slightly gsr-continuous.
2. For every soft clopen subset (G, E) of Y, f^{-1} (G, E) is soft- gsr-closed in X.
3. For every soft clopen subset (G, E) of Y, f^{-1} (G, E) is soft -gsr-clopen in X.

Proof

(1) ⟹ (2) Let (F, E) be soft clopen in Y. Then Y\ (F, E) is soft clopen in Y. Since f is soft slightly gsr-continuous, f^{-1} (Y\ (F, E)) is soft- gsr-open in X. f^{-1} (Y\ (F, E)) = X− f^{-1} (F, E) is soft- gsr-open in X implies f^{-1} (F, E) is soft- gsr-closed in X.

(2) ⟹ (3) Let (F, E) be soft clopen in Y. Then Y\ (F, E) is soft clopen in Y. By (2) f^{-1} (Y\ (F, E)) is soft- gsr-closed in X. Hence f^{-1} (F, E) is soft -gsr-open in X implies f^{-1} (F, E) is soft -gsr-clopen in X.

(3) ⟹ (1) obvious

Theorem 3.3: Every soft slightly continuous function is soft slightly gsr-continuous.
Proof: Let f: (X,τ,E)→(Y,τ′,E) be soft slightly continuous, then f^{-1} (G, E) is soft open in X for each soft clopen subset (G, E) of Y implies that if f^{-1} (G, E) is soft -gsr-open in X for each soft clopen subset (G, E) of Y then is said to be soft slightly gsr –continuous.

Remark 3.4: The converse of the above theorem is not true in general as shown in the following examples.

Example 3.5: Let X=Y={ h_1, h_2, h_3, h_4}, Y = {h_1, h_2, h_3}, E={e_1, e_2}. Let F_1, F_2, ., F_6 and G_1, G_2 are functions from E to P(X) and E to P(Y) are defined as follows:
$F_1(e_1)$ = {h_3}, $F_1(e_2)$ = {h_1}, $F_2(e_1)$ = {h_4}, F_2 (e_2) = {h_2}, $F_3(e_1)$ = {h_3, h_4}, $F_3(e_2)$ = {h_1, h_2}, $F_4(e_1)$ = {h_1, h_4}, $F_4(e_2)$ = {h_2, h_4},
$F_5(e_1)$ = {h_2, h_3, h_4}, $F_5(e_2)$ = {h_1, h_2, h_3},
$F_6(e_1)$ = {h_1, h_3, h_4}, $F_6(e_2)$ = {h_1, h_2, h_4}
Then τ_1 = {$\tilde{\Phi}$, $\tilde{X}$, (F_1,E), ...(F_6,E)} is a soft topological space over X.
$G_1(e_1)$ = {h_1}, $G_1(e_2)$ = {h_1}
$G_2(e_1)$ = {h_2, h_3}, $G_2(e_2)$ = {h_2, h_3}
Then τ_2 = {$\tilde{\Phi}$, $\tilde{X}$, (G_1,E), (G_2,E)} is a soft topological space over Y.
If the function f : (X,τ,E) → (Y,τ′,E) is defined as f(h_1) = h_1, f(h_3) = h_2, f(h_2) = h_3 then f is soft slightly- gsr - continuous but not soft- slightly continuous.

Theorem 3.6: Let (X,τ,E) be a soft gsr - $T_{\frac{1}{2}}$ space. Then the function f: (X,τ,E) → (Y,τ′,E) is soft slightly- gsr-continuous if and only if it is soft slightly continuous.
Proof: Let (G, E) be soft clopen in Y. Since f is soft slightly gsr- continuous, f^{-1} (G, E) is soft- gsr-open in X implies f^{-1} (G, E) is soft open in X. Therefore f is soft slightly continuous. Conversely every soft slightly continuous is soft slightly -gsr-continuous.

Theorem 3.7: Suppose $\tilde{S}$GSRO(X) is soft closed under arbitrary unions. Let f: (X,τ,E) → (Y,τ′,E) be a function. Then f is soft slightly gsr-continuous if and only if for each point *x* ∈ X and each soft clopen set (V, E) containing f(*x*), there exists a soft- gsr-open set (U, E) containing *x* such that f(U,E) $\tilde{\subseteq}$ (V, E).

Proof: Let *x* ∈ X and (V, E) be soft clopen then f(*x*) ∈ (V, E). Since f is soft slightly gsr-continuous, f^{-1} (V, E) is soft -gsr-open in X. If we put (U, E) = f^{-1} (V, E) then *x* ∈ (U,E) and f(U,E) $\tilde{\subseteq}$ (V, E). Conversely Let (V, E) be a soft clopen subset of Y and let *x* ∈ f^{-1} (V, E). Since f(*x*) ∈ (V, E), there exists a soft- gsr-open set (U_x,E) in X containing *x* such that (U_x, E) $\tilde{\subseteq}$ f^{-1} (V, E). We obtain f^{-1} (V, E) = $\cup\{(U_x$,E): *x* ∈ f^{-1} (V, E)}. Thus f^{-1} (V, E) is soft gsr-open.

Definition 3.8: A function f : (X, τ, E) → (Y, τ′, E) is soft-gsr-continuous if f^{-1}(G,E) is soft-gsr-closed(open) in (X, τ, A) for every soft closed(open) set (G, E) in (Y, τ ′,E).

Theorem 3.9: If a function f: (X, τ, E) → (Y, τ′, E) is soft slightly- gsr-continuous and (Y,τ′,E) is soft locally indiscrete, then f is soft -gsr-continuous.

Proof: Let (A, E) be a soft open set in Y. Since Y is soft locally indiscrete, every soft soft open set is soft closed. Since f is soft slightly -gsr-continuous, f^{-1} (A, E) is soft- gsr-open in X. Hence f is soft -gsr - continuous

Theorem 3.10: If a function f: (X,τ,E) → (Y,τ′,E) is soft slightly -gsr-continuous and (X,τ,E) is soft- gsr- $T_{\frac{1}{2}}$ - space then f is soft slightly continuous.

Proof: Let (A, E) be a soft clopen set in Y. By hypothesis f^{-1} (A, E) is soft- gsr-open in X. Since X is soft -gsr- $T_{\frac{1}{2}}$ space, f^{-1} (A, E) is soft open in X. Hence f is soft slightly continuous

Definition 3.11: A function f: (X,τ,E)→(Y,τ′,E) is soft -gsr-irresolute, if f^{-1} (G, E) is soft gsr-open(closed) in (X,τ,E) for every soft- gsr-open(closed) set (G, E) of Y,τ′,E .

Theorem 3.12: Let f: (X,τ,E) → (Y,τ′,E) and g: (Y,τ′,E) → (Z, τ'', E) be functions. Then the following properties hold:
1. If f is soft gsr-irresolute and g is soft slightly- gsr-continuous, then g ∘ f: (X,τ,E) → (Z,τ'',E) is soft slightly-gsr- continuous.
2. If f is soft gsr-irresolute and g is soft gsr- continuous, then g ∘ f: (X, τ, E) → (Z, τ'', E) is soft slightly -gsr-continuous.
3. If f is soft gsr-irresolute and g is soft slightly continuous, then g ∘ f: (X, τ, E) → (Z, τ'', E) is soft slightly- gsr-continuous.

Proof
1. Let (G,E) be soft-clopen in Z, then g^{-1} (G, E) is soft- gsr -open in Y. Since f is soft gsr-irresolute. f^{-1}(g^{-1}(G, E)) is soft -gsr-open in X. But f^{-1}(g^{-1}(G, E)) = $(g\circ f)^{-1}$(G, E) implies that $(g\circ f)^{-1}$(G, E) is soft-gsr-open set in *X*. Thus g∘f is soft slightly- gsr-continuous.
2. Let (F, E) be soft-clopen in Z, then g^{-1} (F, E) is soft -gsr -open(closed) in Y. Since f is soft gsr-irresolute. f^{-1}(g^{-1}(F, E)) is soft-gsr-open (closed) in X. But f^{-1}(g^{-1}(F, E)) = $(g\circ f)^{-1}$(F, E) implies that

$(g \circ f)^{-1}$(F, E) is soft- gsr-open (closed) set in *X*. Thus $g^{o}f$ is soft slighly -gsr-continuous.

3. Let (G,E) be soft-clopen in Z, then g^{-1} (G, E) is soft open in Y. Since f is soft gsr-irresolute. f^{-1}(g^{-1}(G, E)) is soft- gsr-open in X. But f^{-1}(g^{-1}(G, E)) = $(g \circ f)^{-1}$(G, E) implies that $(g \circ f)^{-1}$(G, E) is soft -gsr-open set in *X*. Thus $g^{o}f$ is soft slightly- gsr-continuous.

Definition 3.13: A function f: X $\rightarrow$ Y (where A, B ∈ E) is Soft M-gsr-open if f(F,A) is soft- gsr-open in Y for every soft - gsr -open set (F,A) of X.

Theorem 3.14: Let f: (X,τ,E) → (Y,τ′,E) and g: (Y,τ′,E) → (Z, τ'', E) be functions. If f is soft M-gsr-open surjective and g ^{o} f: (X,τ,E) → (Z,τ'',E) is soft slightly gsr- continuous then g is soft slightly- gsr- continuous.

Proof: Let (A, E) be any soft clopen in Z. Since $g^{o}f$ is soft slightly -gsr-continuous, ($g^{o}f$) (f^{-1}(A, E)) = f^{-1}(g^{-1}(A, E)) is soft gsr-open. Since f is soft M-gsr-open, then f (f^{-1}(g^{-1}(A, E)) = g^{-1}(A, E)) is soft gsr-open in Y. Hence g is soft slightly-gsr-continuous.

Definition 3.15: A space (X,τ,E) is called soft-gsr-connected provided that X cannot be written as the union of two disjoint non-empty soft-gsr-open sets.

Theorem 3.16: If f: (X,τ,E) → (Y,τ′,E) is soft slightly- gsr-continuous surjective function and X is soft gsr-connected, then Y is soft connected.

Proof: Suppose Y is not soft connected. Then there exist non-empty disjoint soft clopen subsets (U, E) and (V, E) of Y such that Y = (U, E) ∪ (V, E). Since f is soft slightly gsr-continuous, we have f^{-1} (U, E) and f^{-1} (V, E) are non-empty disjoint soft gsr-open sets in X. Moreover, f^{-1} (U, E) ∪ f^{-1} (V, E) = X. This shows that X is not soft gsr-connected which is a contradiction. Hence Y is soft connected.

Theorem 3.17: If f is a soft slightly- gsr-continuous function from a soft gsr-connected space X onto any space Y, then Y is not a soft discrete space.

Proof: Suppose that Y is soft discrete. Let (A, E) be a proper nonempty soft open subset of Y. Then , f^{-1} (A, E) is any proper nonempty soft gsr-clopen subset of X, which is a contradiction to the assumption that X is soft gsr-connected. Therefore Y is not a soft discrete space.

Theorem 3.18: A space X is soft gsr-connected, if every soft slightly- gsr-continuous function from a space X into any soft T_0-space Y is constant.

Proof: Suppose that X is not soft gsr-connected. Let every soft slightly- gsr-continuous function from X into any soft T_0-space then Y is constant. Since X is not soft gsr-connected, there exists a proper nonempty soft gsr-clopen subset (A, E) of X. Then f is a non-constant and soft slightly- gsr-continuous such that Y is soft -T_0, which is a contradiction. Hence, X is soft gsr-connected.

4. Soft -gsr-regular space

Definition 4.1: A soft topological space (X, τ, E) is said to be soft - gsr-regular if for every soft- gsr-closed set (G,E) and each point x ∉ (G,E), there exist disjoint soft gsr-open sets (A,E) and (B,E) such that (G,E) $\tilde{\subseteq}$ (A,E), x ∈ (B,E) and (A,E) ∩ (B,E) =ϕ.

Theorem 4.2: Let (X, τ, E) be a soft topological space. If X is a soft- gsr-regular space, then for every point x ∈ X and each soft - open set (G,E) containing x, there exists a soft - open set (F,E) in X such that x ∈ (F,E) $\tilde{\subseteq}$ sgsr-cl (F,E) $\tilde{\subseteq}$ (G,E).

Proof: Let x ∈ X and (G, E) be any soft open set in X such that x ∈ (G,E). Then X − (G,E) is a soft closed set in X such that x ∉ X − (G,E). Since X is soft -gsr-regular space, there exist soft gsr-open sets (F,E), (H,E) in X such that x∈ (F,E), X – (G,E) $\tilde{\subseteq}$ (H,E) and (F,E) ∩(H,E) = ϕ. Now we have (F,E) ∩(H,E) = ϕ. implies sgsr-cl ((F,E) ∩ (H,E)) = ϕ. Also X - (G,E) $\tilde{\subseteq}$ (H,E). Hence sgsr –cl (F,E) $\tilde{\subseteq}$ (G,E). Therefore x ∈ (F,E) $\tilde{\subseteq}$ gsr-cl (F,E) $\tilde{\subseteq}$ (G,E).

Theorem 4.3: If f : X $\rightarrow$ Y is soft continuous, bijective, soft gsr- open function and X is a soft regular space, then Y is soft -gsr-regular.

Proof: Let (F,E) be a soft closed set in Y and let y be a point in a soft space Y in which y ∉ (F,E). Take y=f(x) for some point x in a soft space X. Since f is soft continuous, f^{-1}(F,E) is soft closed set in X such that x ∉ f^{-1}(F,E). (since f(x) ∉ (F,E)). Now, X is soft regular, there exists disjoint soft open sets (A,E) and (B,E) such that x ∈ (A,E) and f^{-1} (F,E) $\subset$ (B,E). i.e. y = f(x) ∈ f(A,E) and (F,E) $\subset$ f(B,E). Since f is soft-gsr-open function, f(A,E) and f(B,E) are soft- gsr-open sets in Y. Since f is soft bijective, f(A,E) $\cap$ f (B,E) = f (A,E) $\cap$ (B,E) = f (ϕ)= ϕ.

$\Rightarrow$ Y is soft gsr-regular.

Theorem 4.4: If f : X $\rightarrow$ Y is soft- gsr-continuous, soft closed injection map and Y is soft regular, then X is soft- gsr-regular.

Proof: Let (F,E) be a soft closed set in X and x ∉ (F,E).

Since f is soft closed injection, f(F,E) is soft closed set in Y such that f(x) ∉ f(F,E).

Now, Y is soft regular, there exists disjoint soft open sets (G,E) and (H,E) such that f(x) ∈ (G,E) and f(F,E) ∈ (H,E).

This implies x ∈ f^{-1}(G,E) and (F,E) $\tilde{\subseteq}$ f^{-1} (H,E).
Since f is soft - gsr-continuous, f^{-1}(G,E) and f^{-1}(H,E) are soft - gsr-open sets in X.
Further, f^{-1}(G,E) ∩ f^{-1} (H,E) = ϕ. Hence X is soft -gsr-regular.

Theorem 4.5: If f: X → Y is soft continuous, soft bijective,soft M- gsr-open function and X is soft - gsr-regular space, then Y is soft- gsr-regular.

Proof: Let (F, E) be a soft closed set in Y and y ∉ (F, E). Take y = f(x) for some point x in a soft space X. Since f is soft continuous and soft bijective, f^{-1}(F, E) is soft closed in X and x ∉
f^{-1}(F, E). Now, since X is soft - gsr-regular, there exists disjoint soft gsr-open sets (A,E) and (B,E) such that x ∈ (A,E) and f^{-1}(F,E) ⊂ (B,E).i.e. y = f(x) ∈ f(A,E) and (F,E) ⊂ f(B,E).
Since f is soft M-gsr-open and soft bijective, f(A,E) and f(B,E) are disjoint soft gsr-open sets in Y. Therefore,Y is soft- gsr-regular .

Theorem 4.6: If f: (X, τ, E) → (Y, τ′, E), is a bijection, soft-gsr- irresolute, soft closed map and Y is soft-gsr-regular space then X is also soft -gsr-regular space.

Proof: Let x ∈ X and (F, E) be any soft closed set in X such that x ∉ (F, E). Since f is a bijection, there exists a point y ∈ Y such that f(x) = y ⇒ x = f^{-1} (y). Also since f is soft closed map, f(F,E) is a soft closed set in Y such that x ∉ (F,E) ⇒ f(x) ∉ f(F,E) ⇒ y ∉ f(F,E). Since Y is soft- gsr-regular space, there exist soft-gsr-open sets (A,E), (B,E) in Y such that y ∈ (A,E), f(F,E) $\tilde{\subseteq}$ (B,E) and (A,E) ∩ (B,E) = ϕ. Since f is soft -gsr-irresolute, f^{-1}(A, E), f^{-1}(B,E) are soft- gsr-open sets in X. Now we have y ∈ (A, E) ⇒ f^{-1} (y) ∈ f^{-1} (A, E) ⇒ x ∈ f^{-1}(A, E); f(F,E) ⊂ (B, E)
⇒ f^{-1}[f(F, E)] ⊂ f^{-1} (B, E) ⇒ (F, E) ⊂ f^{-1} (B, E) and f^{-1} ((A, E) ∩ (B, E)) = f^{-1}(ϕ) ⇒ f^{-1} (A, E) ∩ f^{-1} (B, E) = ϕ, since f is a bijection. Thus, for every point x ∈ X and each soft closed set (F,E) in X such that x ∉ (F,E), there exist soft gsr-open sets f^{-1} (A, E), f^{-1} (B,) in X such that x ∈ f^{-1}(A, E), (F, E) ⊂ f^{-1} (F, E) and f^{-1} (A, E) ∩ f^{-1} (B, E) = ϕ. Hence X is a soft - gsr-regular space.

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